WEST BENGAL STATE UNIVERSITY

B.Sc. Programme 6th Semester Examination, 2022

MTMGDSE04T-MATHEMATICS (DSE2)

LINEAR PROGRAMMING

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

GROUP-A

Full Marks-10

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Is the set $X = \{(x, y) : x^2 + y^2 \le 4\}$ is convex? Justify your answer.

(b) In the following equations find the basic solution with x_3 as the non-basic variable

$$x_1 + 4x_2 - x_3 = 3$$
$$5x_1 + 2x_2 + 3x_3 = 4$$

(c) Find a basic feasible solution of the equations $x_1 + x_2 + x_3 = 8$, $3x_1 + 2x_2 = 18$

(d) Find the extreme points, if any, of the set $S = \{(x, y) : 2x + 3y = 6\}$

(e) Draw the convex hull of the points $(0, 0)$, $(0, 1)$, $(1, 2)$, $(1, 1)$, $(4, 0)$.

(f) Write down the dual of the following L.P.P.:

Maximize $Z = 3x_1 + 5x_2$
Subject to $x_1 + 2x_2 \le 5$
$x_1 - x_2 = 7$
$x_1, x_2 \ge 0$

(g) Determine the position of the point (–1, 2, 5, 3) relative to the hyperplane

$$4x_1 + 6x_2 + x_3 - 3x_4 = 4$$

(h) Find the number of basic feasible solutions of the following L.P.P.:

Maximize $Z = 2x_1 + 3x_2$
Subject to $x_1 + x_2 \ge 2$
$x_1 - x_2 \le 1$
$x_1, x_2 \ge 0$

(i) What is the criterion for no feasible solution in two-phase method?

GROUP-B

Full Marks-40

Answer any *five* questions from the following 8×5 = 40

2. (a) Solve the following L.P.P using graphical method 4

Maximize $Z = 2x_1 + x_2$

Subject to $4x_1 + 3x_2 \le 12$

$4x_1 + x_2 \le 8$

$4x_1 - x_2 \le 8$

$x_1,\ x_2 \ge 0$

(b) Food *X* contains 6 units of vitamin A and 7 units of vitamin B per gram and costs 12 p./gm. Food *Y* contains 8 units of vitamin A and 12 units of vitamin B per gram and costs 20 p./gm. The daily requirements of vitamin A and B are at least 100 units and 120 units respectively. Formulate the above as an L.P.P. to minimize the cost. 4

3. (a) Use Simplex method to solve the L.P.P. 4

Maximize $Z = x_1 + 2x_2 + 4x_3$

Subject to $3x_1 + 5x_2 + 2x_3 \le 6$

$4x_1 + 4x_3 \le 7$

$2x_1 + 4x_2 - x_3 \le 10$

$x_1, x_2, x_3 \ge 0$

(b) Show that the vectors $(1, -2, 0), (3, 1, 2), (5, -1, 4)$ form a basis in E^3. 4

4. (a) Prove that the set of all convex combinations of a finite number of points is a convex set. 5

(b) Find a supporting hyperplane of the convex set 3

$S = \{(x, y): x + 2y \le 4, 3x + y \le 6, x \ge 0, y \ge 0\}$

5. (a) $x_1 = 1,\ x_2 = 1,\ x_3 = 1,\ x_4 = 0$ is a feasible solution of the system of equations 5

$x_1 + 2x_2 + 4x_3 + x_4 = 7$

$2x_1 - x_2 + 3x_3 - 2x_4 = 4$

Reduce the feasible solution to two different basic feasible solutions.

(b) Prove that a hyperplane is a convex set. 3

6. (a) Obtain a basic feasible solution of the following L.P.P. from the feasible solution (2, 3, 1) 4

Maximize $Z = x_1 + 2x_2 + 4x_3$

Subject to $2x_1 + x_2 + 4x_3 = 11$

$3x_1 + x_2 + 5x_3 = 14$

$x_1,\ x_2,\ x_3 \ge 0$

(b) Prove that the intersection of two convex sets is also a convex set. 4

7. (a) Solve by Charnes Big M-method the following L.P.P. 6

Maximize $Z = 4x_1 + x_2$

Subject to $3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 4$

$x_1,\ x_2 \geq 0$

(b) Discuss whether the set of points $(0, 0), (0, 1), (1, 0), (1, 1)$ on the xy-plane is a convex set or not. 2

8. (a) Prove that dual of a dual is a primal. 4

(b) Obtain the dual problem of the following L.P.P. 4

Maximize $Z = -x_1 + 3x_2$

Subject to $2x_1 + x_2 \leq 1$

$3x_1 + 4x_2 \leq 5$

$x_1 + 6x_2 \leq 9$

$x_1,\ x_2,\ x_3 \geq 0$

9. (a) Find the points which generate the convex polyhedron 3

$$S = \{(x_1, x_2) \in E^2 : x_1 + 2x_2 \leq 4,\ x_1 - 2x_2 \leq 2,\ x_1 \geq 0,\ x_2 \geq 0\}$$

(b) Use two-phase method to solve the following L.P.P. 5

Maximize $Z = 3x_1 + 5x_2$

Subject to $x_1 + 2x_2 \geq 8$

$3x_1 + 2x_2 \geq 12$

$5x_1 + 6x_2 \leq 60$

$x_1, x_2 \geq 0$

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within* 1 *hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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